

J 言語 と 固有値問題 (その3)

J and Eigenvalue Problems

VECTOR誌 投稿原稿

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最近の2篇の報告の発展である。

1) 中野嘉弘「J言語と高等数学(固有値問題)」 JAPLA '07 Apr 28

2) 中野嘉弘「J言語と固有値問題 (その2)」 JAPLA '07 May 26

長々しい直接法の J 言語プログラムが、再帰化して、極端に短縮された。

B C S の VECTOR誌 向けの英文投稿を御披露して置く。

「ダニレフスキー法とは何か」の寸見に繋がるか？

J and Eigenvalue problems

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Originally, eigenvalue can be found directly as the latent roots from the polynomial by expanding the eigen (characteristic) determinant.

We expanded the determinant "unconditionally" along the principal diagonal, that is not along a row (nor along a column) as is usually found on the text books of linear algebra.

Generally, the corresponding analytic expressions of the eigen polynomials for the larger matrices than 4x4 are very complicated.

But, our (J language) method gives numerical solutions of larger matrices, iteratively for the degree of upto ten or larger.

If we have no considerations about CPU or memory limit, our procedure may be useful for much larger matrices.

Once the corresponding eigen polynomial is found, the p. function of J language gives the latent roots immediately.

Many other awesome methods of eigenvalue problem (e.g. Jacobi, power, QR, QZ, LZ and so on) must be faded.

Our recursive procedure is tried on J (version J601).

This is not aimed for the shortest runtime, but to illustrate the algorithm of diagonal expansion of the matrices.

```

Naigen=: 3 : 0
:
  Y =. y
  ny =. {. $ Y
  X =. x
  if. X=2 do. Naigen2 y
                                return. end.

  dety =. det y
  my =. minorir y
  Nmy =. (X-1) Naigen each my
  Nmyp =. +/ >Nmy
  e =. Nmyp % (>: i.ny)
x: Nans =. (((_1)^ny)*dety), e  NB. extended precision
)

```

```

Naigen2=: 3 : 0
  dety =. det y
  my =. minorir y
  mdet =. detc my
  e2m =. (-mdet), 1
x: Nans2 =. (dety), e2m
)

```

Example: 10x10 Complex from J.H. Wilkinson (1960)
I1010 NB. data 10x10

```

2j3 3j1 0 0 0 0 0 0 0 0
3j2 _2j_1 1j2 0 0 0 0 0 0 0
5j_3 1j2 2j1 _1j4 0 0 0 0 0 0
2j6 _2j3 3j_1 _4j2 5j5 0 0 0 0 0
1j4 2j2 _3j7 1j5 2j_3 1j6 0 0 0 0
5j_1 0j4 1j5 _8j_1 4j7 7j1 4j_2 0 0 0
5j2 1j4 6j_5 8j4 4j_4 _1j5 3 _4j6 0 0
_4j_3 7j3 1j6 2j_4 3j1 1j2 1j4 6j3 7j_1 0
5 2j2 1j3 1j1 _4j_2 1j6 1j2 2j5 0j1 3j2
5j2 2j6 1j_3 7j4 4j1 7 3j_3 5j_4 6i3 2i5

```

]NI10=. 10 Naigen I1010 NB. J calc
1.75568e8j_6.18637e7 4.01225e7j_7.91003e6

10 1 \$ > {: p. NI10 NB. Eigenvalues
10.7977j8.62338
_4.96687j_8.08712
1.03206j9.29413
8.81131j1.54938
2.38989j7.26808
5.43645j_3.97143
_5.27951j_2.27596
4.16175j3.13751
_1.9352j_3.97509
_2.44755j0.437126

```

minor=: 3 : 0
:
  rx =. 0{x
  cx =. 1{x
  r =. rx-1
  c =. cx-1
  c omite (r omite y)
)

```

```

minorir=: 3 : 0
ny =. 0{$ y
i =. ny
c =. <(i,i) minor y
i =. i-1
while. i > 0 do.
  ci =. <(i,i) minor y
  c =. c, ci
  i =. i-1
end.
c
)

```

additional J functions:

```
each=: &.>
det=: -/. *
detc=: 3 : 0
    +/> (det each y)
)

omitr=: 3 : 0
    :
    (<<<x) {y
)
omite=: 3 : 0
    :
    (<<<x) {"1 y
)
```

In writing this report, the author found old books commenting similar idea.

H.W.Turnbull, A.C. Aitken: " An Introduction to the Theory of Canonical Matrices " Dec 1931, 2nd Ed. Nov 1944, St.Andrews Edinburgh , p.41:

" the characteristic function
where p_r is the sum of the diagonal minors of order r of the matrix A , and p_n is the determinant of A itself. (Invariants p.98). "

Here "Invariants" is the book of the same authors ,"The Theory of Determinants, Matrices, and Invariants 1928".

But the hints how to get the sums of diagonal minors are given nowhere. To express the hint may be difficult in analytic forms.

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Danilevskiĭ A.: O čislennom rešenii vekovogo uravneniya. Mat. Sbornik Bd. 2 (1937). S. 169 - 171

Rudolf Zurmühl (O.Proffesor an der Technischen Universität Berlin):

'Matrizen und ihre technischen Anwendungen' 1964

Vierte Neubearbeitete Auflage, Springer-Verlag

