Ｊ言語　と　固有値問題　　（その３）

J and Eigenvalue Problems

VECTOR誌　投稿原稿

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最近の２篇の報告の発展である。
1)　中野嘉弘「J 言語と高等数学（固有値問題）」　JAPLA '07 Apr 28
2) 中野嘉弘「J 言語と固有値問題　（その2）」　JAPLA '07 May 26
長々しい直接法の J 言語プログラムが、再帰化して、極端に短縮された。
ＢＣＳの VECTOR誌 向けの英文投稿を御披露して置く。
「ダニレフスキー法とは何か」の寸見に繋ながるか？

J and Eigenvalue problems

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Originally, eigenvalue can be found directly as the latent roots from the polynomial by expanding the eigen (characteristic) determinant.
We expanded the deteminant "unconditionally" along the principal diagonal, that is not along a row (nor along a column) as is usually found on the text books of linear algebra.

Generally, the corresponding analytic expressions of the eigen polynomials for the larger matrices than 4x4 are very complicated.
But, our (J language) method gives numerical solutions of larger matrices, iteratively for the degree of upto ten or larger.

If we have no considerations about CPU or memory limit, our procedure may be useful for much larger matrices.
Once the corresponding eigen polynominal is found, the p. function of J language gives the latent roots immediately.
Many other awesome methods of eigenvalue problem (e.g. Jacobi, power, QR, QZ, LZ and so on) must be faded.

Our recursive procedure is tried on J (version J601).
This is not aimed for the shortest runtime, but to illustrate the algorithm of diagonal expansion of the matrices.

```
Naigen=: 3 : 0
    :
    Y =. y
   ny=. {. $ Y
   X=. x
  if. X=2 do. Naigen2 y
                                    return.  end.
   dety =. det y
   my=. minorir y
   Nmy=: (X-1) Naigen each my
    Nmyp=.  +/ >Nmy
    e=. Nmyp % ( >: i.ny)
x:  Nans =. (((_1)^ny)*dety), e     NB. extended precision            )

  Naigen2=: 3 : 0
       dety=. det y
      my=. minorir y
         mdet=. detc my
         e2m=. (-mdet), 1
   x:  Nans2=. (dety), e2m
  )
```

Example: 10x10 Complex   from J.H. Wilkinson (1960)

```
  I1010       NB.   data  10x10

 2j3    3j1    0      0     0     0    0     0    0    0
 3j2   _2j_1   1j2    0     0      0    0    0    0    0
 5j_3   1j2    2j1   _1j4   0      0    0    0     0    0
 2j6   _2j3    3j_1  _4j2   5j5    0    0    0     0    0
 1j4    2j2   _3j7    1j5   2j_3   1j6  0     0     0   0
 5j_1   0j4    1j5   _8j_1  4j7    7j1  4j_2  0     0    0
 5j2    1j4    6j_5   8j4   4j_4  _1j5  3    _4j6   0    0
_4j_3   7j3    1j6    2j_4  3j1    1j2  1j4   6j3   7j_1  0
 5      2j2    1j3    1j1  _4j_2   1j6  1j2   2j5   0j1  3j2
 5j2    2j6    1j_3   7j4   4j1    7    3j_3  5j_4

  ]NI10=. 10 Naigen I1010     NB. J calc
1.75568e8j_6.18637e7 4.01225e7j_7.91003e6

   10 1 $  > {: p. NI10   NB. Eigenvalue
 10.7977j8.62338
_4.96687j_8.08712
 1.03206j9.29413
 8.81131j1.54938
 2.38989j7.26808
 5.43645j_3.97143
_5.27951j_2.27596
 4.16175j3.13751
 _1.9352j_3.97509
_2.44755j0.437126
```

```
minor=: 3 : 0
     :
     rx=.0{x
     cx=.1{x
     r=. rx-1
     c=. cx-1
    c omitc (r omitr y)
    )

 minorir=: 3 : 0
   ny=. 0{$ y
   i=.ny
     c=.<(i,i) minor y
   i=.i-1
  while. i >0 do.
    ci=.< (i,i) minor y
   c=. c, ci
    i=.i-1
  end.
     c
 )
```

aditional J functions:

```
each=: &.>
det=: -/ . *
detc=: 3 : 0
    +/> (det each  y)
)

omitr=: 3 : 0
    :
    (<<<x) {y
    )
omitc=: 3 : 0
    :
    (<<<x) {"1 y
    )
```

In writing this report, the author found old books commenting similar idea.

H.W.Turnbull, A.C. Aitken: " An Introduction to the Theory of Canonical Matrices " Dec 1931, 2nd Ed. Nov 1944, St.Andrews Edinburgh , p.41:

" the characteristic function ....
where pr is the sum of the diagonal minors of order r of the matrix A, and pn is the determinant of A itself. (Invariants p.98). "

Here "Invariants" is the book of the same authors ,"The Theory of Determinants, Matrices, and Invariants 1928".

But the hints how to get the sums of diagonal minors are given nowhere. To express the hint may be dificult in analytic forms.

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Danilevskiĭ A.: O čislennom rešenii vekovogo uravneniya. Mat. Sbornik Bd. 2 (1937). S. 169 - 171

Rudolf Zurmühl (O.Proffesor an der Technischen Universität Berlin):

'Matrizen und ihre technischen Anwendungen' 1964

Vierte Neubearbeitete Auflage, Springer-Verlag